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The spin-symmetry of the quark model

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Abstract. Corrections to the exact heavy–quark symmetry results are expected to come from the inverse powers of the heavy-quark mass. We show, by an explicit calculation using the quark model, that the breaking of the spin symmetry is suppressed by other kinematic effects even when the quark masses are not heavy.

1 Introduction

The heavy–quark symmetry, which appears in the heavy–quark limit, gives exact results for the decays of heavy hadrons [1]. Due to the heavy–quark symmetry all form factors in the heavy–to–heavy type of decays such as $B \to D^{(*)} e\bar{\nu}_e$ ($D^{(*)} = D$ or D^*) can be related, in the heavy–quark limit, to a single universal function called the Isgur–Wise function. The Isgur–Wise function is of nonperturbative origin and has been of great interest to both theoretical and experimental studies. In the heavy–quark symmetry limit, the decoupling of the heavy–quark spin with other light fields leads to symmetry relations among hadronic matrix elements. The corrections to the symmetry are expected to come from inverse powers of the mass of the heavy quark. It has been unclear how well these would extrapolate to heavy–to–light quark decays, although presumably the charm quark might still be heavy enough.

Many of these symmetries were anticipated in some versions of the constituent quark model but there has not been an estimate of how much of these were an artifact of the quark model, and of the choice of wave functions. In a paper using the relativistic quark model [2], we found that the breaking of the spin symmetry among hadronic form factors is small even for heavy–to–light quark decays. In this paper, we show explicitly, using the same model, that there exist kinematic effects that would also suppress the breaking of spin symmetry. In fact, the quark model keeps the spin–symmetry rules remarkably well even for a wide range of masses.

We first recapitulate some aspects of spin symmetry for mesons in the heavy–quark limit. For a pseudoscalar meson $P(Q\bar{q})$ with heavy constituent quark Q, the spin of Q decouples from all other light fields in P [3]. We can therefore construct the spin operator S_Q^Z for Q such that in the heavy m_Q limit

$$S_Q^Z |P(Q\bar{q})\rangle = \frac{1}{2} |V_L(Q\bar{q})\rangle \,, \tag{1}$$

where $V_L(Q\bar{q})$ is the longitudinal component of a vector meson with the same quark content as P. In practice, the spin symmetry in (1) can be transformed into identities between the hadronic matrix elements, and thus some form factor relations, for $H \to P$ and $H \to V_L$, where $H(h\bar{q})$ is a pseudoscalar meson with a heavy quark h. Using the relation [3] $[S_Q^Z, A^0 + A^3] = (1/2)(V^0 + V^3)$ for the currents $V_\mu = \bar{Q}\gamma_\mu h$ and $A_\mu = \bar{Q}\gamma_\mu\gamma_5 h$ of the transition $h \to Q$, it can be shown that (1) leads to the following identity between the hadronic matrix elements for $H \to V_L$ and $H \to P$,

$$\langle V_L | A^0 + A^3 | H \rangle = \langle P | V^0 + V^3 | H \rangle \,. \tag{2}$$

In $|P\rangle$ and $|V_L\rangle$ the spatial momentum of the quark Q is defined in the z–direction for the Q spinor to be an eigenstate of S_Q^Z. The spatial momenta of P and V_L should therefore also be defined in the z–direction such that the correction to the spin symmetry is of the order of Λ/m_Q, where Λ is the internal energy scale of P and V_L.

2 Kinematics

In this paper, we consider the breaking of the spin symmetry coming from a finite quark mass m_Q by directly calculating, in particular, the hadronic matrix elements in (2). We use the relativistic quark model formulated in the infinite momentum frame (or equivalently, the light–front quark model) [2,4–6]. We first define the ratio of the matrix elements in (2) as

$$\rho(q^2) = \frac{\langle P(k') | V^0 + V^3 | H(p) \rangle}{\langle V_L(k) | A^0 + A^3 | H(p) \rangle} \,, \tag{3}$$

so that $1 - \rho$ represents a measure of the breaking of the spin symmetry since $\rho = 1$ in the heavy–quark limit.

The ratio ρ is a function of momentum transfer such that $\langle V_L|A^0 + A^3|H\rangle$ and $\langle P|V^0 + V^3|H\rangle$ are evaluated at the same q^2. This allows us to evade the usual problem of kinematic discrepancies when we come to consider a number of different final states. Since the spatial momenta of P and V_L are in the z–direction, we define the function ρ in a frame where the parametrization of the momenta in the z–direction is given by

$$\begin{aligned} p^\mu &= (E_H; 0, 0, p^z)\,, \\ k^\mu &= (E_V; 0, 0, k^z)\,, \\ k'^\mu &= (E_P; 0, 0, k'^z)\,. \end{aligned} \tag{4}$$

The vector and scalar masses m_V and m_P are different for finite m_Q, so V_L and P will not carry the same momentum even though the initial state H has the same p^z. We write the momenta k^z and k'^z in terms of the frame parameter p^z through the condition $q^2 = (p-k)^2 = (p-k')^2$. The general parametrization in (4) includes the particular case of the infinite momentum frame in which $k^z = k'^z = p^z = P$ where $P \to \infty$ and $q^2 = 0$. It is important to note that the function ρ, when calculated in the infinite momentum frame, is defined at $q^2 = 0$ only[1]. This is also the point of maximum recoil, which is usually difficult to treat in a non–relativistic quark model, since a large amount of energy is given to the outgoing particle.

The mass–shell conditions for p, k, and k' give the following constraints on the momenta in (4)

$$\frac{m_V}{m_P}\left(\frac{E_P + k'^z}{E_V + k^z}\right) = \frac{w - \sqrt{w^2-1}}{w' - \sqrt{w'^2-1}}\,, \tag{5}$$

where $w = p\cdot k/(m_H m_V)$ and $w' = p\cdot k'/(m_H m_P)$. The ratio $(E_P + k'^z)/(E_V + k^z)$ in (5) is therefore invariant for the frame defined in (4) and is a function of q^2 through the relation $q^2 = (p-k)^2 = (p-k')^2$.

It can be shown from the covariant expansion of the hadronic matrix elements [7] that $\langle P(k')|V^0 + V^3|H(p)\rangle/(E_P + k'^z)$ and $\langle V_L(k)|A^0 + A^3|H(p)\rangle/(E_V + k^z)$ are invariant with respect to the frame defined in (4). Using the kinematic constraint in (5), it is easy to see that the function $\rho(q^2)$ is an invariant quantity. The matching of p^z in $H \to V_L$ and $H \to P$ of ρ is the only choice that would lead to this invariance. In the definition of $\rho(q^2)$, there is an ambiguity coming from the fact that the ranges of q^2 are usually quite different in $H \to V_L$ and $H \to P$. We will therefore consider the value of ρ at $q^2 = 0$ only. The hadronic matrix elements in (3) can be calculated reliably using the relativistic quark model formulated in the infinite momentum frame. So at $q^2 = 0$, we can write

$$\rho(0) = \rho^{IMF}\,, \tag{6}$$

where ρ^{IMF} is calculated in the infinite momentum frame.

3 Symmetry breaking

A brief introduction to the relativistic quark model in the infinite momentum frame can be found in Refs. [2,5,6]. In the relativistic quark model, the wave function for the ground state meson $M(Q\bar{q})$ is given by

$$|M(\mathbf{k})\rangle = \sqrt{2}\int d\mathbf{p}_Q \sum_{\sigma\,\bar\sigma} \Psi^{Jm_J}_{M,\,\sigma\bar\sigma}|Q(\mathbf{p}_Q,\sigma)\bar{q}(\mathbf{k}-\mathbf{p}_Q,\bar\sigma)\rangle\,, \tag{7}$$

where $\mathbf{k} = P\hat{\mathbf{z}}$ is the spatial momentum of the meson M, $\mathbf{p}_Q = (\mathbf{p}_T, xP)$ and $\mathbf{p}_{\bar q} = \mathbf{k} - \mathbf{p}_Q = (-\mathbf{p}_T, (1-x)P)$ are those of the quarks Q and $\bar q$, respectively, in the infinite momentum frame. Here, Ψ is the momentum wave function for the $Q\bar q$ bound state. It has the separable form into the spin and orbital parts as $\Psi^{Jm_J}_{M,\,\sigma\bar\sigma} = R^{Jm_J}_{M,\,\sigma\bar\sigma}\,\phi_M$, where the expressions for $R^{Jm_J}_{M,\,\sigma\bar\sigma}$ and ϕ_M can be found in [2,6].

In the relativistic quark model, it is shown below that ρ^{IMF} is a function of the mass ratios r_Q, $r_{\bar q}$ and r_Λ, where

$$r_Q = \frac{m_Q}{m_h}\,, r_{\bar q} = \frac{m_{\bar q}}{m_h}\,, r_\Lambda = \frac{\Lambda}{m_h}\,.$$

Thus, we have $\rho(0) = \rho^{IMF}(r_Q, r_{\bar q}, r_\Lambda)$. The parameter Λ determines the internal energy scale of the meson and should be in the order of Λ_{QCD}. The dependence on r_Λ appears only in the momentum wave function and actually there could be a separate Λ for each of the mesons resulting in three different parameters. We take them all to be equal here. The kinematic region of interest for r_Q, $r_{\bar q}$, and r_Λ is such that $0 < r_Q, r_{\bar q}, r_\Lambda \le 1$.

In the relativistic quark model, the matrix elements in (3) are calculated in terms of momentum wave function integrals $\mathcal{I}_1$ and $\mathcal{I}_2$ as,

$$\begin{aligned} \langle P(k')|V^0 + V^3|H(p)\rangle &= 4P(2\pi m_h^2)\mathcal{I}_1\,, \\ \langle V_L(k)|A^0 + A^3|H(p)\rangle &= 4P(2\pi m_h^2)\mathcal{I}_2\,, \end{aligned} \tag{8}$$

so that

$$\rho^{IMF} = \frac{\mathcal{I}_1}{\mathcal{I}_2}\,. \tag{9}$$

The momentum wave function integrals $\mathcal{I}_1$ and $\mathcal{I}_2$ are given by

$$\mathcal{I}_1 = \int_0^1 dx \int_0^\infty dy\, y\, \phi_H\, \phi_P \frac{\alpha_0(1, r_{\bar q})\alpha_0(r_Q, r_{\bar q}) + y^2}{d_0(1, r_{\bar q}) d_0(r_Q, r_{\bar q})} \tag{10}$$

and

$$\begin{aligned} \mathcal{I}_2 = \int_0^1 dx \int_0^\infty dy\, y\, \phi_H\, \phi_V \Big\{ &\alpha_0(1, r_{\bar q})\alpha_1(r_Q, r_{\bar q})\alpha_2(r_Q, r_{\bar q}) \\ &+ y^2\left[\alpha_1(r_Q, r_{\bar q}) - \alpha_2(r_Q, r_{\bar q}) + \alpha_0(1, r_{\bar q})\right]\Big\}\Big/ \\ &\Big\{ d_0(1, r_{\bar q})\, d_1(r_Q, r_{\bar q})\, d_2(r_Q, r_{\bar q}) \Big\}\,, \end{aligned} \tag{11}$$

[1] For decays of a pseudoscalar meson to another pseudoscalar meson there are usually two form factors while for the decay of a pseudoscalar to a vector there are four independent form factors. (For a definition see Ref. [2]). However, in the infinite momentum or light–front frame, at $q^2 = 0$, we have the following much simpler connection for ρ:

$$\rho^{IMF} = \frac{F_1^{H\to P}(0)}{A_0^{H\to V}(0)}\,.$$

with the definitions

$$\alpha_0(r_Q, r_{\bar{q}}) = x r_{\bar{q}} + (1-x) r_Q ,$$
$$\alpha_1(r_Q, r_{\bar{q}}) = r_Q + x M_0(r_Q, r_{\bar{q}}) ,$$
$$\alpha_2(r_Q, r_{\bar{q}}) = r_{\bar{q}} + (1-x) M_0(r_Q, r_{\bar{q}}) ,$$
$$M_0(r_Q, r_{\bar{q}}) = \sqrt{\frac{r_Q^2 + y^2}{x} + \frac{r_{\bar{q}}^2 + y^2}{1-x}} ,$$
$$d_0(r_Q, r_{\bar{q}}) = \sqrt{\alpha_0^2(r_Q, r_{\bar{q}}) + y^2} ,$$
$$d_1(r_Q, r_{\bar{q}}) = \sqrt{\alpha_1^2(r_Q, r_{\bar{q}}) + y^2} ,$$
$$d_2(r_Q, r_{\bar{q}}) = \sqrt{\alpha_2^2(r_Q, r_{\bar{q}}) + y^2} .$$

Although the form of (9 − 11) may look somewhat unfamiliar since it is written in term of ratios of masses, it can readily be checked against the more standard notation given in [2,5]. It is easy to see that the factors $\alpha_{0,1,2}$, $d_{0,1,2}$, and M_0 above are all positively defined in the kinematic regions of x and y so that the wave function integrals $\mathcal{I}_1$ and $\mathcal{I}_2$, the two matrix elements in $\rho(0)$ itself, are both positive. Thus, the ratio $\rho(0)$ is positive and the breaking $1-\rho(0)$ is always less than 1. The sign of $\mathcal{I}_2$ requires some justification as there is a term $M_0(r_Q, r_{\bar{q}})(1-x)^2(r_Q - y^2)$ on expanding out the numerator of the integrand. Since $< y^2 > \sim r_\Lambda^2$ and r_Q is greater than $< y^2 >$, it is then clear that this term should also be positive unless possibly for light-to-light quark decay. We have shown numerically below that this is still not the case.

We may write the orbital wave functions ϕ_H and $\phi_{P,V}$ in terms of a Gaussian function $\phi(r_Q, r_{\bar{q}}, r_\Lambda)$ such that $\phi_H = \phi(1, r_{\bar{q}}, r_\Lambda)$ and $\phi_{P,V} = \phi(r_Q, r_{\bar{q}}, r_\Lambda)$. The expression for $\phi(r_Q, r_{\bar{q}}, r_\Lambda)$ is given by [2,5,6,8]

$$\phi(r_Q, r_{\bar{q}}, r_\Lambda) = N\sqrt{\frac{dz}{dx}} exp\left(-\frac{1}{2}(y^2+z^2)/r_\Lambda^2\right) , \quad (12)$$

where N is a normalization factor that is canceled out in ρ^{IMF}, and

$$z = \left(x - \frac{1}{2}\right) M_0(r_Q, r_{\bar{q}}) - \frac{(r_Q^2 - r_{\bar{q}}^2)}{2M_0(r_Q, r_{\bar{q}})} .$$

In [2], it has been pointed out that the scaling behavior of the meson decay constant f_M in the heavy–quark limit imposes a constraint on the orbital wave function. The Gaussian wave function in (12) is shown to satisfy the scaling law $1/\sqrt{m_Q}$ of f_M in the heavy m_Q limit.

In the numerical analysis of $\rho(0)$, it is convenient to set $r_\Lambda = r_{\bar{q}}$ and vary $r_{\bar{q}}$ and r_Q within the kinematic region of $0 < r_{\bar{q}}, r_Q \le 1$. The spectator quark $\bar{q}$ is thus considered to be a light quark with $m_{\bar{q}} = \Lambda \sim \Lambda_{\mathrm{QCD}}$. The heaviness of the decaying quark is defined relative to Λ through the ratio $r_{\bar{q}}$. In case of a heavy quark decaying to a heavy or to a light quark, the corresponding regions for $r_{\bar{q}}$ and r_Q are such that $r_{\bar{q}}$ is small and r_Q varies between 1 and 0. For a light quark that decays to another light quark, we look instead at the region where $r_{\bar{q}}$ and r_Q are both close to 1.

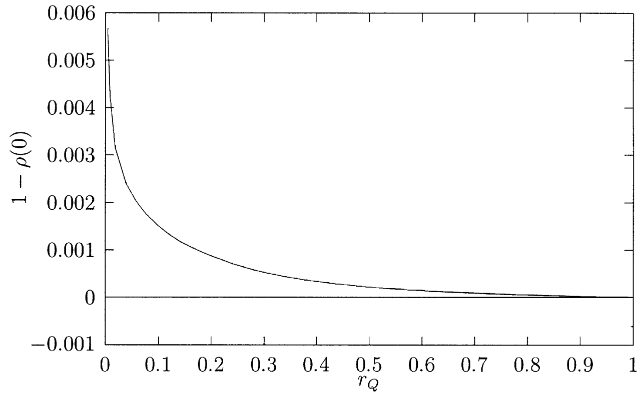

Fig. 1. The plot of $1-\rho(0)$ using the Gaussian orbital wave function. The plot is for heavy–to–heavy and heavy–to–light decays with a quark mass ratio $r_{\bar{q}} = r_\Lambda = 0.001$ and where r_Q varies within the range of $0 < r_Q \le 1$. (r_Q refers to the ratio $r_Q = m_Q/m_h$ in the quark decay $h \to Q$).

When $r_{\bar{q}} = r_Q = 1$, the symmetry breaking is calculated to be $1-\rho(0) = -0.33$, using the Gaussian wave function in (12). Thus, the heavy–quark symmetry does not apply in this case, as expected.

Toy model

Before we deal with physical masses we use a toy model for a *very* heavy quark decaying to a heavy or to a light quark. This will allow us to see, closely to a strict heavy-quark limit, the mass effect of the recoiling quark to the symmetry in the relativistic quark model. In Fig. (1), we show the plot of $1-\rho(0)$, where the mass ratios $r_{\bar{q}}$ and r_Λ are very small (here they are taken to be $r_{\bar{q}} = r_\Lambda = 0.001$ for which the decaying quark is around the top mass scale) and the variation of r_Q is within the range $0 < r_Q \le 1$. We can see that $1-\rho(0)$ is positive except near $r_Q = 1$. As it will be discussed below, $1-\rho(0)$ will be positive for the full range of r_Q when $r_{\bar{q}}$ and r_Λ are exactly zero, or m_h is infinitely heavy in the strict limit. Since $\rho(0)$ represents the ratio of form factors $\frac{F_1^{H\to P}(0)}{A_0^{H\to V}(0)}$ as given in footnote 1, we can check this with calculations using recent updated quark model parameters [9], where it seems to hold even for finite, physical masses. From the Gaussian form of the wave function in Eq.(12), one expects $\langle y^2 \rangle = r_\Lambda^2$. According to the integral expression for ρ^{IMF} in (9), it is easy to see that $1-\rho(0) \to 0$ for a very heavy decaying quark since $\langle y^2 \rangle \to 0$ in the integrands of $\mathcal{I}_1$ and $\mathcal{I}_2$. From the figure we see that the symmetry breaking is less than 1% overall ($|1-\rho(0)| < 0.01$ compared with $\rho(0) = 1$ in the strict limit) in the heavy–quark limit (small $r_{\bar{q}} = 0.001$). The mass effect of m_Q (heavy quark effective theory) can be seen clearly as the breaking of the spin symmetry gradually decreases as we move from small r_Q (heavy–to–light decays) towards the region of larger r_Q (heavy–to–heavy decays). However, even at very small r_Q, it is remarkable that the symmetry breaking is less than 0.6%, showing that the heavy limit is relatively unaffected by finite mass corrections.

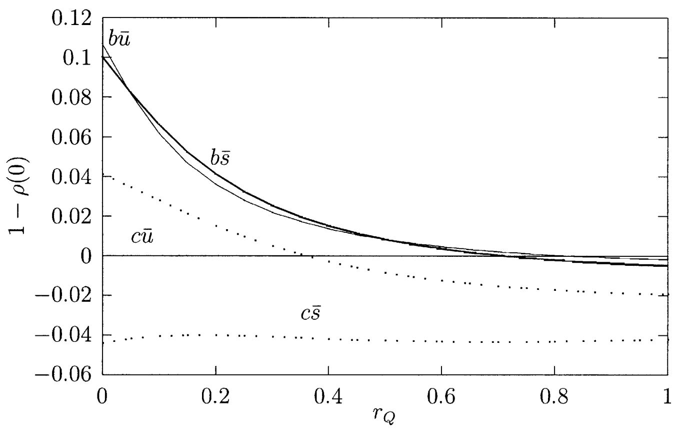

Fig. 2. The plot of $1-\rho(0)$ with physical spectator quark mass ratios of $r_{\bar{q}}(b\bar{u}) = m_{\bar{u}}/m_b = 0.06$ and $r_{\bar{q}}(b\bar{s}) = m_{\bar{s}}/m_b = 0.1$ for heavy b decays, and $r_{\bar{q}}(c\bar{u}) = m_{\bar{u}}/m_c = 0.2$ and $r_{\bar{q}}(c\bar{s}) = m_{\bar{s}}/m_c = 0.3$ for heavy c decays.

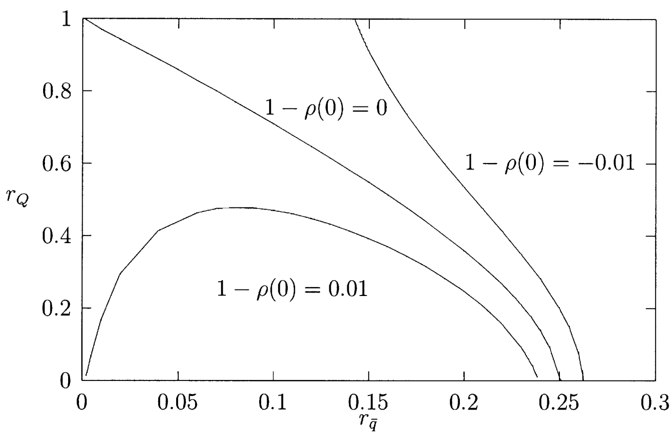

Fig. 3. The corresponding values of $r_{\bar{q}}$ and r_Q for which $1-\rho(0) = 0$ and $1-\rho(0) = \pm 0.01$.

Physical mass results

In Fig. (2), we show the plot of $1-\rho(0)$ away from the strict heavy–quark limit when physical quark mass ratios are used. For heavy–quark decays with spectator quark such as $\bar{u}$ or $\bar{s}$, we have $r_{\bar{q}}(b\bar{u}) = m_{\bar{u}}/m_b = 0.06$ and $r_{\bar{q}}(b\bar{s}) = m_{\bar{s}}/m_b = 0.1$, respectively, in case of b decays, and for c decays, we have $r_{\bar{q}}(c\bar{u}) = m_{\bar{u}}/m_c = 0.2$, and $r_{\bar{q}}(c\bar{s}) = m_{\bar{s}}/m_c = 0.3$. In the figure, the breaking of the spin symmetry is shown to be less than 10% for heavy b and c quark decays. In the strictly heavy–quark limit of $r_{\bar{q}} \to 0$ as indicated in Fig. (1), the mass effect of m_Q is clearly seen as the suppression of the symmetry breaking increases, or $1-\rho(0)$ is getting smaller toward zero, with $r_Q = m_Q/m_h$. Notice, however, that in cases of the finite physical spectator mass ratios above, the breaking $1-\rho(0)$ passes through zero at a recoil mass below its heaviest limit of $r_Q = 1$. This indicates that the largest suppression of the symmetry breaking appears at a point other than the heaviest recoil mass in contrary to the expectation from the mass suppression of the heavy quark effective theory.

In the cases of b decays ($b\bar{u}$ and $b\bar{s}$) and the decay of charm with a non–strange spectator ($c\bar{u}$) as shown in Fig. (2), the mass effect for the suppression of spin–symmetry breaking is still seen for wide range of r_Q where $1-\rho(0)$ is positive. In the region where $1-\rho(0)$ is negative, however, the mass suppression no longer follows as the magnitude of $1-\rho(0)$ actually increases with r_Q. In spite of that, the size of the breaking is still small. This suggests two things: (1) the recoil mass effect in the heavy-quark effective theory is no longer dominating, and (2), the spin symmetry, while not occurring in the expected way, is nevertheless not badly broken even when the recoil mass is light. We show below that when the decaying quark has finite mass there are indeed kinematic factors other than the recoil mass effects that govern the size of the symmetry breaking. The breaking $1-\rho(0)$ is also shown to be suppressed by an overall factor of $r_\Lambda^2/r_{\bar{q}}$ regardless of the recoil mass.

As shown in Fig. (2), the symmetry breaking $1-\rho(0)$ reaches zero and becomes negative at smaller value of r_Q when the mass ratio $r_{\bar{q}}$ is larger. In the strict heavy-quark limit of $r_{\bar{q}} \to 0$, the point of zero breaking appears at the heaviest recoil mass of $r_Q = 1$ revealing predominantly the mass suppression effect in $1-\rho(0)$. For physical spectator quark mass ratio $r_{\bar{q}}$, the symmetry limit of zero breaking appears at lower recoil mass r_Q below the heaviest limit when the decaying quark is relatively less heavier. This indicates that kinematic effects are more pronounced when $r_{\bar{q}}$ is larger since the mass suppression becomes less obvious. As discussed below, the kinematic effect goes like $1-r_Q$ in the strict limit for which it behaves in the same way as the mass suppression. When $r_{\bar{q}}$ is larger, the kinematic effect deviates from such behaviour obscuring the overall mass suppression. Numerically, when $r_{\bar{q}} \geq 0.251$, $1-\rho(0)$ is negative for all r_Q and the mass effect is totally obscured. A different type of behavior then enters for the charm decay with a strange spectator ($c\bar{s}$) where the breaking of the symmetry is rather constant and about 4%.

In Fig. (3), we show the corresponding values of $r_{\bar{q}}$ and r_Q for which the spin symmetry holds exactly, viz., $1-\rho(0) = 0$. The point of zero breaking is shown to lie within the region where $r_{\bar{q}}$ is small ($r_{\bar{q}} \leq 0.251$) and the decaying quark is heavy. As shown in the figure, the zero of $1-\rho(0)$ appears at smaller r_Q as the mass of the decaying quark becomes less heavy (larger $r_{\bar{q}}$). Also shown in the figure are the plots for which the spin–symmetry breaking is about 1% that is $|1-\rho(0)| = 0.01$. It can been seen that a large portion of the possible phase space of r_Q and $r_{\bar{q}}$ is within the region where $|1-\rho(0)| \leq 0.01$.

Kinematic suppression

How does the kinematic effect that gives rise to *zero* symmetry breaking in $1-\rho(0)$ come about? To see this most clearly and to show that it is indeed a kinematic effect, we expand for small r_Λ^2 the kinematic terms of $\mathcal{I}_1$ and $\mathcal{I}_2$ in y^2 (as $< y^2 > \sim r_\Lambda^2$) and integrate over y. In the Taylor expansion, we relax the condition of $r_\Lambda = r_{\bar{q}}$ adopted

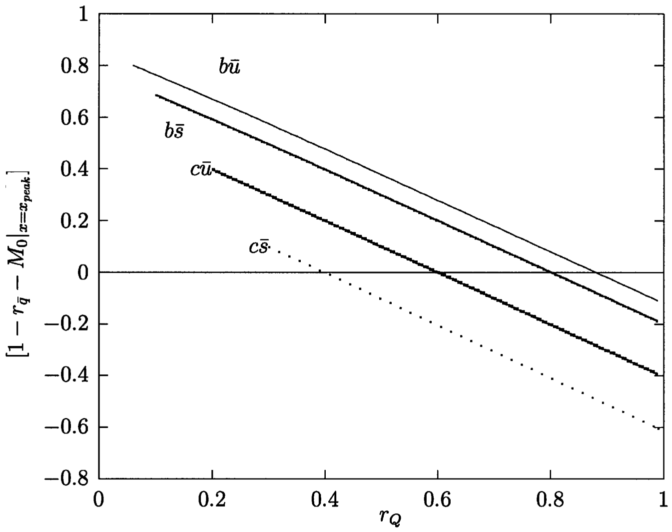

Fig. 4. The plot of the kinematic suppression $[1 - r_{\bar{q}} - M_0(r_Q, r_{\bar{q}})|_{x=x_{peak}}]$ with physical spectator quark mass ratios of $r_{\bar{q}}(b\bar{u}) = m_{\bar{u}}/m_b = 0.06$, $r_{\bar{q}}(b\bar{s}) = m_{\bar{s}}/m_b = 0.1$, $r_{\bar{q}}(c\bar{u}) = m_{\bar{u}}/m_c = 0.2$, and $r_{\bar{q}}(c\bar{s}) = m_{\bar{s}}/m_c = 0.3$.

in the numerical analysis and impose only the condition that $r_\Lambda \sim r_{\bar{q}}$. This gives the leading term of the symmetry breaking $1 - \rho(0)$ in r_Λ^2 as

$$1 - \rho(0) = r_\Lambda^2 \frac{\int_0^1 dx \varphi(1, r_{\bar{q}}, r_\Lambda)\varphi(r_Q, r_{\bar{q}}, r_\Lambda)\Omega}{\int_0^1 dx \varphi(1, r_{\bar{q}}, r_\Lambda)\varphi(r_Q, r_{\bar{q}}, r_\Lambda)}, \qquad (13)$$

where

$$\Omega = \frac{2(1-x)[1 - r_{\bar{q}} - M_0(r_Q, r_{\bar{q}})]}{\alpha_0(1, r_{\bar{q}})\alpha_1(r_Q, r_{\bar{q}})\alpha_2(r_Q, r_{\bar{q}})} .$$

Here the wave function $\varphi(r_Q, r_{\bar{q}}, r_\Lambda) = \sqrt{\frac{dz}{dx}} exp(-(z^2)/2r_\Lambda^2)$ is the z–part of ϕ in (12) and the term $M_0(r_Q, r_{\bar{q}})$ has now $y^2 = 0$. The overall power suppression of order $r_\Lambda^2/r_{\bar{q}}$ for $1-\rho(0)$ as stated previously becomes clear since the leading term in the Taylor expansion is of order r_Λ^2, and the term $(1-x)/\alpha_0\alpha_1\alpha_2$ in Ω behaves like $\mathcal{O}(1/r_{\bar{q}})$ only as described below. The kinematic effect leading to zero breaking in $1-\rho(0)$ can readily be seen from the $[1 - r_{\bar{q}} - M_0(r_Q, r_{\bar{q}})]$ term in Ω for which kinematic suppression in addition to the mass effect occurs when $M_0(r_Q, r_{\bar{q}})$ is close to $1 - r_{\bar{q}}$. Since $m_h^2 M_0^2(r_Q, r_{\bar{q}}) = (p_Q + p_{\bar{q}})^2$ is the momentum sum of the recoiling $Q\bar{q}$, kinematic suppression in $1 - \rho(0)$ occurs when $(p_Q + p_{\bar{q}})^2 \approx (m_h - m_{\bar{q}})^2$.

For $r_Q > r_{\bar{q}}$, the contribution to $1 - \rho(0)$ in the integrals of (13) is dominated by the Gaussian peak at $x = x_{peak} \cong 1 - r_{\bar{q}}/\sqrt{r_Q}$ coming from the wave function overlap of $\varphi(1, r_{\bar{q}}, r_\Lambda)\varphi(r_Q, r_{\bar{q}}, r_\Lambda)$. The integrals can be approximated by setting $x = x_{peak}$ in the integrands so that $1 - \rho(0) = r_\Lambda^2 \Omega|_{x=x_{peak}}$. This gives

$$1 - \rho(0) = \frac{r_\Lambda^2[\,1 - r_{\bar{q}} - M_0(r_Q, r_{\bar{q}})|_{x=x_{peak}}\,]}{r_{\bar{q}} r_Q (1 + \sqrt{(r_Q)})^2}, \qquad (14)$$

for which the symmetry breaking $1 - \rho(0)$ reveals both $1/m_Q$ mass suppression through the $r_Q = m_Q/m_h$ term in the denominator and kinematic suppression through the $[1 - r_{\bar{q}} - M_0(r_Q, r_{\bar{q}})|_{x=x_{peak}}]$ term in the numerator. The overall power suppression of order $r_\Lambda^2/r_{\bar{q}}$ for $1 - \rho(0)$ as stated above can readily be seen from (14). The kinematic suppression $[1 - r_{\bar{q}} - M_0(r_Q, r_{\bar{q}})|_{x=x_{peak}}]$ in (14) behaves differently from mass suppression as shown in Fig. (4) for which it reaches zero prior to the heaviest limit of $r_Q = 1$ and stays negative thereafter. Also, the point of zero appears at lower recoil mass r_Q from the heaviest limit when $r_{\bar{q}}$ is larger. This renders the mass suppression in $1 - \rho(0)$ to be obscured by the kinematic effect (more pronounced for larger $r_{\bar{q}}$), and the overall mass behaviour of $1 - \rho(0)$ deviates from $1/m_Q$ suppression when $r_{\bar{q}}$ is further away from the strict limit. In the strict heavy-quark limit of $r_{\bar{q}} \to 0$, it is easy to show that the kinematic suppression $[1 - r_{\bar{q}} - M_0(r_Q, r_{\bar{q}})|_{x=x_{peak}}] \to 1 - r_Q$ for which it reaches zero at $r_Q = 1$. The symmetry breaking $1 - \rho(0)$ will stay positive for all r_Q and the mass suppression behaviour will remain.

The quantity $\rho(0)$ has the following physical meaning:

$$|\rho(0)|^2 = \frac{(m_H^2 - m_V^2)^3}{(m_H^2 - m_P^2)^3} \frac{d\Gamma(H \to Pl\bar{\nu})/dq^2|_{q^2=0}}{d\Gamma(H \to V_L l\bar{\nu})/dq^2|_{q^2=0}} . \qquad (15)$$

This allows a test of these results to be made by considering the q^2 spectrum for the semileptonic decays $H(h\bar{q}) \to P, V_L(Q\bar{q})$. The size of $\rho(0)$ for particular values of r_Q and $r_{\bar{q}}$ can now be measured. Repeating this for the different semileptonic decay channels of H, the dependence of $\rho(0)$ with r_Q and $r_{\bar{q}}$ can also be determined.

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